THE TRANSFINITE ELEMENT METHOD FOR MODELING MMIC DEVICES

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ABSTRACT

A new numerical procedure called the transfinite element method is employed in conjunction with the planar waveguide model to analyze MMIC devices. By using analytic basis functions together with finite element approximation functions in a variational technique, the transfinite element method is able to determine the fields and scattering parameters for a wide variety of stripline and microstrip devices.

INTRODUCTION

The analysis of the MMIC devices is often performed by using a planar waveguide approximation. This procedure requires two steps [1] [2] [3]

1. Approximate the actual three-dimensional MMIC device with an equivalent N-port planar waveguide model.
2. Solve for the electromagnetic fields and scattering matrix coefficients in the equivalent planar waveguide model.

We propose here a new method for the second of these steps that is considerably more efficient and more accurate than the existing alternatives. The method is based on the transfinite element procedure first proposed by the authors for the solution of unbounded electrostatics problems [4] and later extended to the solution of electromagnetic scattering problems [5]. Unlike the eigensolution procedure reported in [2] that requires that a set of orthonormalized eigenmodes be determined for the planar waveguide, the transfinite element method is deterministic and hence is much more efficient. And, unlike the finite difference time domain method of [3], the transfinite element method reported here is time harmonic and thus eliminates the need for expensive numerical time integration.

THE PLANAR WAVEGUIDE MODEL

The first part of the modeling procedure is to transform the MMIC device into a planar circuit that can be solved by two-dimensional analysis. This is accomplished by replacing the actual dimensions of the MMIC device with effective dimensions for an equivalent planar waveguide. This operation is different with striplines than it is with microstrip:

1. In stripline circuits, the dominant propagating mode is TEM, for which effective dimensions are easily calculated by using quasi-static analysis or by using the empirical formula of reference [1].
2. In microstrip, the dominant mode is non-TEM and the field pattern thus varies with frequency. In the low frequency limit, the TEM approximation can be used to construct an equivalent planar waveguide model. Formulas that account for dispersion may then be used to modify the effective dimensions at higher frequencies. In a few cases, successful formulas for this purpose are reported in the literature [6]; however, as yet, no such transformation procedure is available for arbitrary geometries.

THE TRANSFINITE ELEMENT METHOD

The transfinite element method is an extension of the finite element procedure in which analytical functions are used as basis functions in certain, special regions of the solution domain. In the planar circuit problem, ordinary finite element approximation functions are used to represent the field in the neighborhood of the discontinuity; the special regions for transfinite element analysis are the semi-infinite waveguide sections leading to the ports of the waveguide discontinuity. In these semi-infinite regions, modal analysis is used to obtain basis functions that satisfy the governing equation exactly.

Six steps are required to apply the transfinite element method to planar N-port microwave circuits:

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1. Divide the problem region into a discontinuity region R_d and semi-infinite port regions R_{p_i} separated by port reference planes P_i. Let R_d contains all the discontinuities and inhomogenities. Approximate the solution in R_{p_i} as

$$E_i^p = \delta_i \exp^{-\gamma_{i0} y} + \sum_{j=0}^{M} a_{ij} \cos(\frac{j\pi x}{d_i}) \exp^{\gamma_{ij} y} \quad (1)$$

$$\delta_i = \begin{cases} 1 & \text{if } i = 1 \text{ (input port)} \\ 0 & \text{otherwise} \end{cases}$$

where d_i is the width of port i, γ_{ij} is the propagation constant for mode j in port i.

2. Write the solution in R_d in terms of standard finite element approximation functions.

3. Impose continuity of the electric field across P_i by requiring that the field approximations in R_d and in R_{p_i} be identical at the finite element nodes.

4. Evaluate the bilinear functional

$$\begin{aligned} \mathcal{B}(u,v) &= \int_{R_d} (\nabla v^\star \cdot \nabla u - k^2 v^\star u)\, d\Omega \\ &- \sum_i \int_{P_i} v^\star \frac{\partial u}{\partial n}\, d\Gamma - \sum_i \int_{R_{p_i}} v^\star (\nabla^2 u + k^2 u)\, d\Omega \end{aligned} \quad (2)$$

where $\star$ denotes the complex conjugate.

5. Impose the stationarity of this functional to give a symmetric matrix equation.

6. Solve the resulting symmetric matrix equation for the field.

The novelty and power of the transfinite element analysis procedure is derived from the following:

- The finite element method models any local region efficiently. Adding analytical approximation functions to the procedure extends the domain of approximation to infinity.
- The approximation functions (1) used in the variational procedure are in fact just the first M solutions for the modes in an infinite waveguide. Hence, that last integral in (2) vanishes.
- Derivative continuity along the port reference planes P_i does not need to be imposed explicitly but is generated automatically by the natural boundary conditions inherent to the variational procedure.

RESULTS

A general purpose computer program has been developed to model MMIC devices using the transfinite element method [7]. The program is graphics oriented, easy to use and fast. It allows planar waveguide junctions of any shape to be analysed quickly and easily.

Shown in Figure 1(a) is a two-port stripline filter with a circular disk. The characteristic impedance of both ports is 50 Ω, the substrate height is $2h = 0.64cm$ and the relative dielectric constantis 2.4. To solve this problem with the transfinite element method, we first convert to the equivalent planar waveguide model with the effective dimensions shown in Figure 1(b). This shape is then discretized by using triangular finite elements, and solved by means of the transfinite element method. The transmission coefficient computed by means of this procedure is plotted in Figure 1(c). As shown, good agreement exists between these results and published experimental data [6].

It should be noted that the curve in Figure 1(c) was produced by using the adaptive spectral response modeling procedure in [8]. The squares on the abscissa of this graph correspond to the frequencies actually employed in the computation.

Figure 2(a) shows a microstrip offset step discontinuity with eccentricity δ with substrate height $h = 0.15cm$. The corresponding planar waveguide model at low frequency is shown in Figure 2(b). The frequency-dependent effective parameters are calculated by using the formulas in reference [9]. A comparison of the transmission coefficient computed by transfinite element method and by generalized scattering matrix method [10] is given in Figure 2(c). Again, good agreement confirms the validity of the present method.

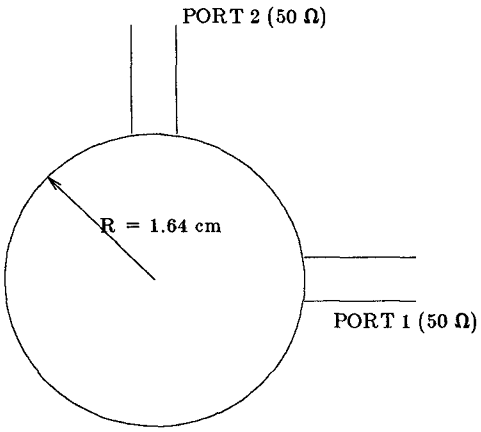

Figure 1(a): Geometry of a stripline disk band-elimination filter coupled to two 50 Ω striplines.

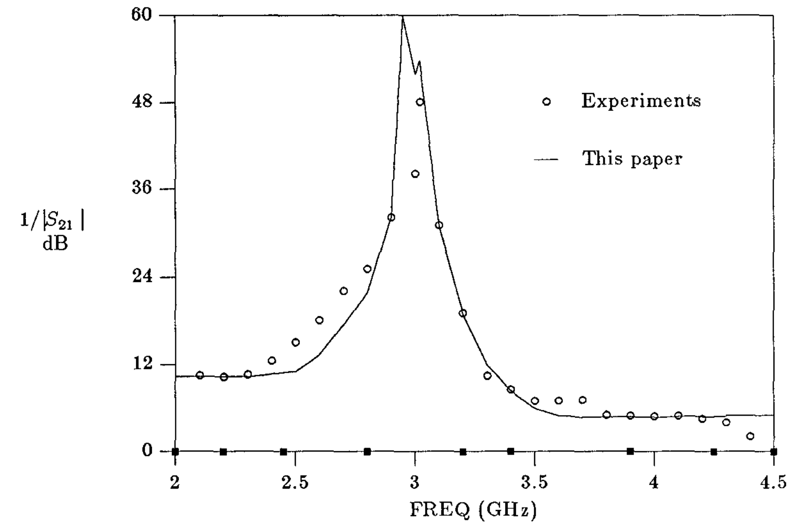

Figure 1(c): Transfinite element method transmission parameter for the band-elimination filter compared with experimental values

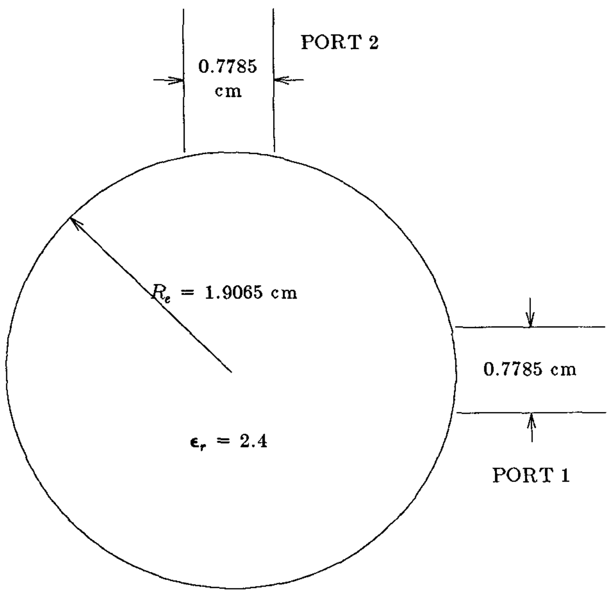

Figure 1(b): Planar waveguide model for the stripline disk in 1(a)

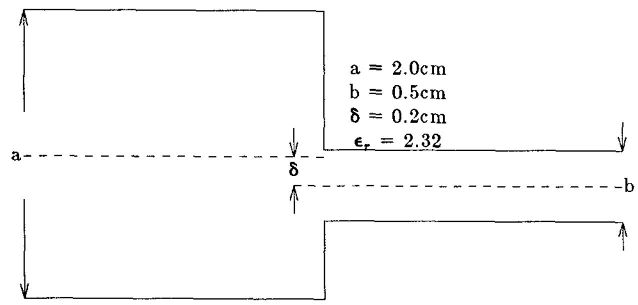

Figure 2(a): Microstrip offset step discontinuity with a substrate height $h = 0.15cm$

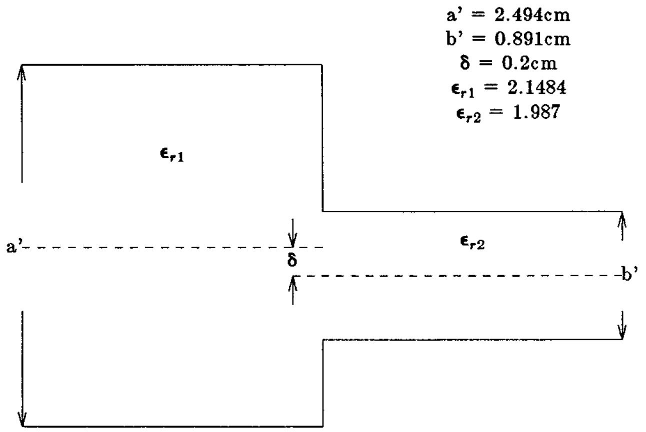

Figure 2(b): Planar waveguide model at low frequency for Figure 2(a)

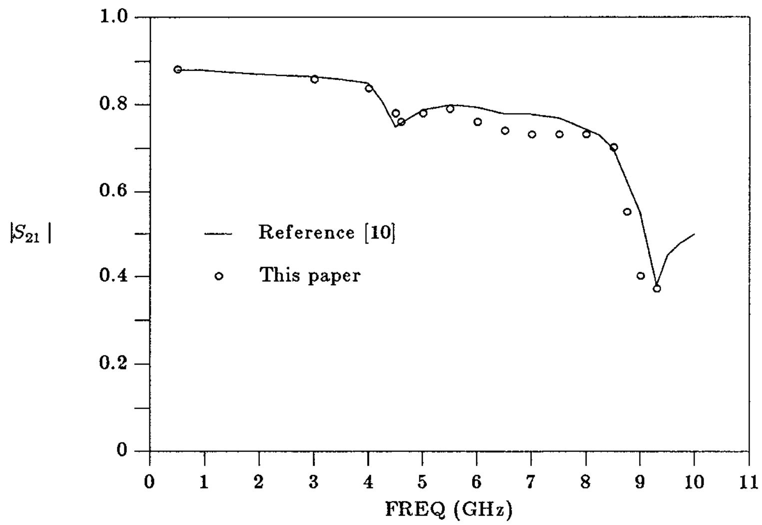

Figure 2(c): Comparison of transmission parameter for the circuit in Figure 2(a) with reference [10]

References

[1] T. Okoshi and T. Miyoshi, "The planar circuit - An approach to microwave integrated circuitry." *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 245-252, Apr. 1972.

[2] R. Sorrentino, "Planar Circuits, Waveguide Models, and Segmentation Method." *IEEE trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1057-1066, Oct. 1985.

[3] W. K. Gwarek, "Analysis of an Arbitrarily-Shaped Planar Circuit - A Time-Domain Approach." *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1067-1072, Oct. 1985.

[4] J. F. Lee and Z. J. Cendes, "Transfinite elements: A highly efficient procedure for modeling open field problems." *J. Appl. Phys.*, vol. 61, pp. 3913-3915, Apr. 1987.

[5] J. F. Lee and Z. J. Cendes, "The Transfinite Element Method for Computing Electromagnetic Scattering from Arbitrary Lossy Cylinders." *IEEE AP-S International Symposium Digest.*, AP03-5, pp. 99-102, June 1987.

[6] R. R. Bonetti and P. Tissi, "Analysis of Planar Disk Networks." *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 471-477, July 1978.

[7] "Maxwell Microwave IC Computer Program Users Guide." *Ansoft Corporation*, Pittsburgh, PA, 1987.

[8] J. F. Lee and Z. J. Cendes, "An Adaptive Spectral Response Modeling Procedure for Multi-Port Microwave Circuits." *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 1240-1247, Dec. 1987.

[9] K. C. Gupta, R. Garg and I. J. Bahl, "Microstrip Lines and Slotlines." *Artech House*, 1979.

[10] T. S. Chu and T. Itoh, "Generalized Scattering Matrix Method for Analysis of Cascaded and Offset Microstrip Step Discontinuities." *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 280-284, Feb. 1986.